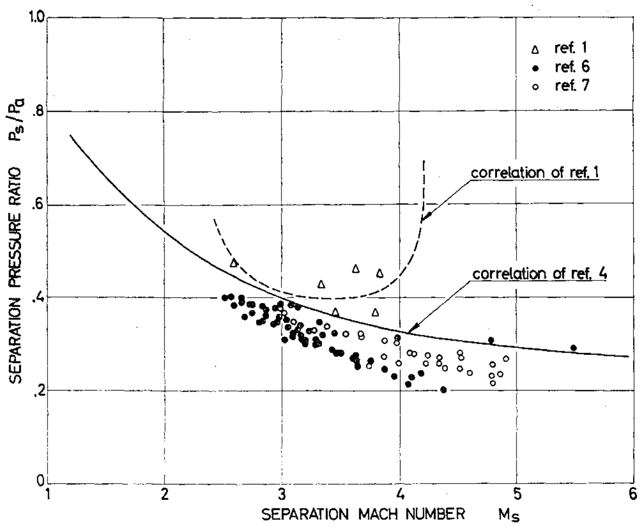

Fig. 1 Separation pressure ratio for contoured nozzles.

nozzles. The overall trend is similar to that for straight-walled nozzles. The correlation suggested in Ref. 1, transformed to the coordinates of Fig. 1, is shown also. It clearly is not adequate.

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Offset-Aim Target Seeker Technique for Interplanetary Ballistic Trajectories

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THE determination of the exact initial conditions for a precise interplanetary ballistic trajectory is a problem met frequently in astrodynamic studies. A common approach is to linearize the equations of motion, generate the elements of a coefficient matrix, solve for a set of changes, and iterate until a satisfactory trajectory is attained. This note presents an iteration scheme, more effective and involving fewer computations than the matrix approach.

Received March 19, 1963. This work was sponsored in part by the NASA George C. Marshall Space Flight Center, under Contract NAS 8-2469, and in part by the Independent Research Program of Lockheed Missiles and Space Company.

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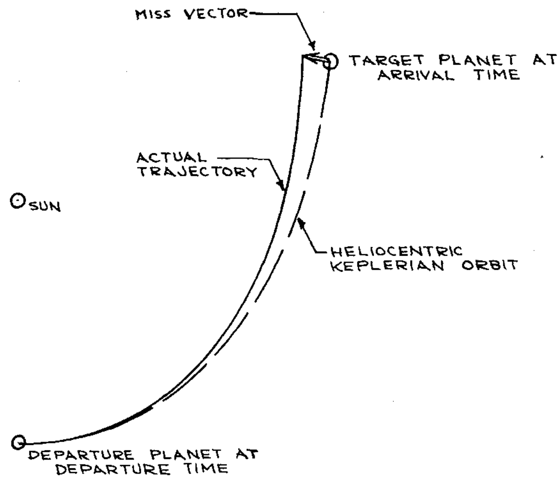

Fig. 1a Trajectory geometry; first pass.

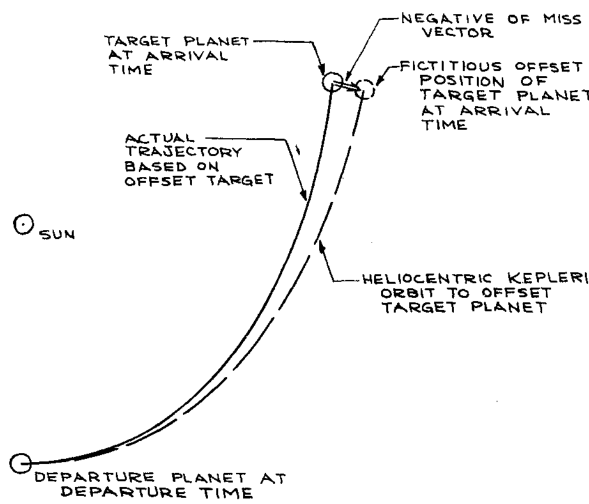

Fig. 1b Trajectory geometry; iterative pass.

The nature of the problem to be solved is characterized as follows: determine the initial conditions for a precise interplanetary ballistic flight that leaves and arrives on specified dates; the trajectory departs along a vertical launch‡ or from a parking orbit and arrives with a vertical impact or into a specified orbit about the target planet; and the trajectory integration includes the usual solar system perturbations.

Description of the Technique

The iteration scheme employs an offset-aim technique to converge on the required initial conditions. Approximate launch conditions are computed from the two-body heliocentric unperturbed Keplerian orbit equations. With these launch conditions an exact trajectory, including all perturbations, is calculated to the vicinity of the target planet, and the miss distance vector is noted; this step is depicted in Fig. 1a. Then, the arrival end of the heliocentric Keplerian orbit is offset by the negative of this miss vector, and the launch conditions are recomputed for the next exact trajectory integration as shown in Fig. 1b. The process is repeated, and the initial conditions are improved until the miss vector is reduced to a tolerable size. The miss distance vector at the target planet is the miss measured relative to the incoming planetocentric hyperbolic asymptote, similar to the vector $\bar{B}$ of Kizner.[1] This vector behaves in a nearly linear manner

‡ Although a vertical launch admittedly is not realistic for flight planning, it is useful for parametric studies.

with corrections to the aiming point and thus contributes to the rapid convergence of the technique. Use of the periapse vector is not recommended since this vector changes in a highly nonlinear fashion. The miss vector is reckoned in a convenient way by opening up the trajectory integration mesh size when about five to ten spheres of influence away from the target planet and jumping to the final time point in one step. Such a large step size eliminates nearly all the *terminal* gravity effects of the target planet but retains all the other perturbative action.

After the target planet has been intercepted satisfactorily, a small step-size trajectory is computed to determine the precise arrival conditions. At this point, time-of-arrival errors may be noted. These errors are treated by similarly offsetting the target time of arrival.

Implementing the Technique

In carrying out the iteration technique, several computational tools and methods have proved useful and effective. First, a subroutine to solve the Keplerian equations, given the relative position of and the time of travel between the two terminal planets, must be available. This subroutine must yield continuous results. The one based on Lambert's theorem developed by Breakwell, Gillespie, and Ross[2] works extremely well and rapidly. Second, the ephemeris routine also must produce continuous data for the planets, with consistent position and velocity information. The semianalytic program devised by the authors[3] has proved to be well adapted to this problem. Third and finally, a conversion from the heliocentric initial velocity found from the Keplerian equations to the planetocentric launch conditions is needed. The technique of subtracting the heliocentric velocity of the launch planet from that of the initial Keplerian orbit and interpreting this relative velocity as the launch hyperbolic excess velocity has led simply and directly to the determination of the launch conditions. Obviously, the errors in this approximation are corrected by the offset-aim technique.

Examples

A variety of one-leg interplanetary trips were run to test the offset-aim technique. Voyages from the Earth to Mars, Earth to Venus, and Venus to Earth of relatively long and short durations were included. Values of the excess hyperbolic velocity ranged from the lows associated with near-Hohmann transfers to the highs associated with transfers that crossed the planetary orbits at fairly large angles. Departures from the Earth were either from a specified parking orbit, in particular, one that resulted from a boost from Cape Canaveral into the Atlantic Missile Range, or from a vertical launch; departures from Venus were only from a vertical launch. Only vertical impacts at the target planet were considered. The final runs were continued until the miss distance was reduced to the order of 10 to 100 miles and the error in time of arrival to about 5 min or less.

The description and results of the exemplary trips are presented in Table 1. Through use of the offset-aim technique, the initial miss distances were reduced by three to almost five orders of magnitude. Impact, defined as a miss distance smaller than the radius of the target planet, was achieved in one or two iterations in nearly all cases. Only one run, no. 3, had a final miss distance greater than 100 miles; on the other hand, the companion run, no. 4, launched from a parking orbit, converged to a miss distance of 44 miles (47×10^{-8} a.u.). For the class of trips considered and the accepted standards of accuracy, the upper limit on the number of iterations needed to converge on both position and time of arrival apparently depended primarily upon the average hyperbolic excess speed at the terminal planets; the runs to the heavier planets, Earth and Venus, usually required a few more iterations than those to Mars because of the larger gravity-induced time errors that built up during the terminal portion of the trajectory. As more experience is acquired in employing this technique, the number of iterations will be reduced by combining more efficiently the runs employed to reduce the position and time errors.

Table 1 Offset-aim target seeker examples

Trip no.	Termini[a]	Duration, days	Kind of trajectory	Kind of launch[c]	Average hyperbolic excess speed, EMOS[d]	Initial miss, 10^{-5} a.u.	Final miss, 10^{-8} a.u.[e]	Final time error, min.	No. of iterations for first impact	Final no. of iterations
1	⊕ → ♂	90	Short	V	0.317	28	24	5.0	1	2
2		90	Short	O	0.317	223	20	4.9	1	3
3		570	Long	V	0.265	887	139	0.17	2	4
4		570	Long	O	0.265	790	47	0.04	2	5
5		300	L. opt.[b]	V	0.135	147	12	0.66	2	6
6		300	L. opt.	O	0.135	399	62	0.17	2	6
7		178	S. opt.[b]	V	0.134	285	20	2.8	2	8
8		777	Circumsolar	V	0.730	275	34	1.2	1	2
9	⊕ → ♀	60	Short	V	0.250	33	38	1.6	1	6
10		300	Long	V	0.249	609	15	0.13	1	7
11		300	Long	O	0.272	2112	27	0.30	3	9
12		165	L. opt.	V	0.101	84	36	6.6	2	7
13		131	S. opt.	V	0.095	212	43	5.4	2	9
14		300	High excess speed	V	0.694	130	31	4.0	1	3
15	♀ → ⊕	60	Short	V	0.298	25	32	1.1	1	3
16		300	Long	V	0.235	744	44	0.22	2	6
17		160	L. opt.	V	0.099	532	42	4.4	2	9
18		135	S. opt.	V	0.102	311	67	5.1	2	7
19		300	High excess speed	V	0.558	251	14	0	1	4

[a] Planetary symbols: ⊕ Earth; ♂ Mars; ♀ Venus.
[b] L. opt. (S. opt.): long (short) optimum, i.e., the orbital transfer with locally minimum departure hyperbolic excess speed having a transfer time slightly greater (less) than a 180° transfer orbit (refer to the speed contour charts of Ref. 2).
[c] O means orbital and V means vertical.
[d] EMOS is the Earth mean orbital speed. 1 EMOS = 18 495 statute miles/sec = 29.765 km/sec.
[e] 10^{-8} a.u. = 0.92893 statute miles = 1.4950 km.

Conclusions

The offset-aim target seeker technique is a rapid, effective method for obtaining the launch conditions for fixed time-of-travel interplanetary trajectories. It lends itself well to the employment of simplified methods because it corrects for any biases so introduced. However, accurate, repeatable, and continuous ephemeris, trajectory integration, and Keplerian programs must be available for use with it.

For the examples given, an unperturbed Keplerian orbit is, as expected, a fairly good approximation to the exact interplanetary trajectory that includes the gravitational effects of the solar system planets. Such an orbit provides an excellent first estimate and computational tool for carrying out the offset-aim target seeker method.

Additional applications of this technique are being considered for the following: 1) trajectories between the Earth and the moon; 2) trajectories that satisfy constraints more complex than those just described; 3) multi-leg nonstop trajectories that leave the Earth, pass one or more of the other planets, and return to the Earth; and 4) calculation of interplanetary guidance corrections.

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Spreading of Liquid-Surface Jets Supported by Buoyancy Forces

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THE spreading of two-dimensional free jets in laminar and turbulent flows has been studied by H. Schlichting, W. Bickley, W. Tollmien, and H. Goertler and the results are summarized in Ref. 1. In this note, the free jets results will be extended to liquid-surface jets that are lighter than the bulk of the liquid due to a temperature difference.

The static pressure at a point inside the liquid, relative to that in the gas space above, may be written as

$$p(x,y) = g \int_0^y \rho(x,y)dy \tag{1}$$

where ρ is the liquid density and g is the local gravitational acceleration (in the y direction). Differentiating Eq. (1) with respect to x and introducing the volumetric expansion coefficient β, it can be shown for small temperature differences that

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = g\,\beta\,\frac{\partial}{\partial x}\int_0^y (T - T_\infty)dy \tag{2}$$

where T is the local temperature and T_∞ is the temperature outside the mixing layer.

Introducing the following dimensionless variables $U = u/u_1$, $V = v/u_1$, $\Theta = (T - T_\infty)/(T_1 - T_\infty)$, $X = u_1x/\nu$, $Y = u_1y/\nu$, $Pr = C_p\rho\nu/k$, and $a = g\beta(T_1 - T_\infty)\nu/u_1^3$, the boundary-layer equation for laminar flows can be written as

$$(\partial U/\partial X) + (\partial V/\partial Y) = 0 \tag{3}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = a\frac{\partial}{\partial X}\int_0^Y \Theta dY + \frac{\partial^2 U}{\partial Y_2} \tag{4}$$

$$U\frac{\partial \Theta}{\partial X} + V\frac{\partial \Theta}{\partial Y} = \frac{1}{Pr}\frac{\partial^2\Theta}{\partial Y^2} \tag{5}$$

where u_1 and T_1 are the initial velocity and temperature of the liquid surface jet; u and v are the horizontal and vertical components of velocity; and ν, k, C_p, Pr are kinematic viscosity, thermal conductivity, specific heat at constant pressure, and Prandtl number, respectively. The boundary conditions are

$$\text{At } Y = 0: V = 0 \qquad \frac{\partial U}{\partial Y} = 0 \qquad \frac{\partial \Theta}{\partial Y} = 0$$

$$\text{As } Y \to \infty: U = \Theta \to 0$$

It is assumed that the free surface is adiabatic and that the drag by the gas above the free surface is zero. The momentum and energy integrals may be obtained by integration of Eqs. (4) and (5), yielding

$$\int_0^\infty U^2 dY - a\int_0^\infty \int_0^Y \Theta dY dY = \frac{u_1h}{\nu} - \frac{a}{2}\left(\frac{u_1h}{\nu}\right)^2 \equiv M \tag{6}$$

$$\int_0^\infty U\Theta dY = \frac{u_1h}{\nu} \equiv E \tag{7}$$

where h is the initial width of the surface jet. Equations (6) and (7) also define M and E.

A similar solution for the system of Eqs. (3–7) with the boundary conditions given does not exist. However, a similar solution for laminar free jets ($a = 0$) exists with $\psi \sim X^{1/3}F(\eta)$, $U \sim X^{-1/3}F'(\eta)$, and $\eta = Y/X^{2/3}$, where ψ is the stream function and $F(\eta)$ is a function of the similarity variable η. From Eq. (7), $\Theta \sim X^{-1/3}L(\eta)$, where $L(\eta)$ is a function of η, in order to satisfy the requirement that the energy integral is independent of X. For $a \neq 0$, one may consider the following series expansions:

$$F(\eta, aX) = \frac{\psi}{X^{1/3}} = F_0(\eta) + aXF_1(\eta) + (aX)^2F_2(\eta) + \ldots \tag{8}$$

$$L(\eta, aX) = \Theta X^{1/3} = L_0(\eta) + aXL_1(\eta) + (aX)^2L_2(\eta) + \ldots \tag{9}$$

With $U = (\partial\psi/\partial Y)_X$, $-V = (\partial\psi/\partial X)_Y$, substituting into Eqs. (3–7), collecting terms, and equating the coefficients of the powers of (aX) to zero, yields

$$3F_0''' + (F_0')^2 + F_0''F_0 = 0 \tag{10}$$

$$3F''' + (3n-2)\int_0^\eta L_{n-1}\,d\eta - 2\eta L_{n-1} = \sum_{k=0}^{n}[(3k-1)\times F_k'F_{n-k}' - (3k+1)F_kF''_{n-k}] \qquad n \geqq 1 \tag{11}$$

$$\frac{3}{Pr}L_0'' + F_0'L_0 + F_0L_0' = 0 \tag{12}$$

$$\frac{3}{Pr}L_n'' = \sum_{k=0}^{n}[(3k-1)L_kF'_{n-k} - (3k+1)F_kL'_{n-k}] \qquad n \geqq 1 \tag{13}$$

$$\int_0^\infty (F_0')^2 d\eta = M = \frac{u_1h}{\nu} - \frac{a}{2}\left(\frac{u_1h}{\nu}\right)^2 \tag{14}$$

Received June 10, 1963.

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